Concrete slab deflection

STRAP calculates the linear elastic deflection of a concrete slab based on the gross cross-section moment-of-inertia. However the actual slab deflections are much greater due to several important factors:

- cracking
- reinforcement ratio
- time-dependant non-linear factors, such as creep and shrinkage.

The STRAP results module has an option to calculate the deflection using a method which takes into account these factors. The method is an empirical one based on an "effective" moment-of-inertia approach and it important to understand that this method is not an exact one.

The method calculates an "effective" (reduced) moment-of-inertia that is a function of the ratio of the actual moment to the cracking moment of the element.

**Eurocode 2:**

\[ I_e = 0.5 \left( \frac{M_{cr}}{M} \right)^2 I_g \cdot \left( 1 - 0.5 \left( \frac{M_{cr}}{M} \right)^2 \right) I_{cr} \leq I_g \]

**ACI 318:**

\[ I_e = \left[ \frac{M_{cr}}{M} \right]^4 I_g \cdot \left( 1 - \left[ \frac{M_{cr}}{M} \right]^4 \right) I_{cr} \leq I_g \]

where the fourth power is used as suggested by Branson for continuous integration.

for both codes:

- \( I_e \) = effective moment-of-inertia
- \( I_g \) = gross moment-of-inertia, including reinforcement
- \( I_{cr} \) = cracked moment-of-inertia
- \( M \) = service moment
- \( M_{cr} \) = cracking moment

STRAP calculates the effective moment-of-inertia and for each element in both direction and then solves the model again using the reduced stiffness values.

The total deflection \( a_t \) is the sum of the immediate deflection \( a_i \) from all service loads and the long-term deflection \( a_l \) from the sustained service loads, therefore different stiffness values are used for immediate and long-term deflection calculations based on the value of \( M \) derived from the loads applied; the user must define different load combinations for immediate and long-term loads.

**Geometry**

- click the new model icon
- select and click

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**Nodes:**
- define the eleven corner nodes that form the slab contour:

**Elements:**
- click `Elements` and then select in the side menu.
- click `OK` in the following menu.
- select the eleven corner nodes (in the order they are numbered) to define the floor contour and close by selecting the first node again.
- select `End contour definition`.
- click `OK` in the following menu to accept the default mesh parameters; the program creates and displays the floor slab.
- click `Property` in the side menu and click and highlight Property 1 in the table.
- Define thickness = 200 mm and $E = 30,000$ MPa ($30 \times 10^6$ kN/m$^2$)

**Restraints:**
Define pinned supports at the nodes as shown in the following drawing:

**Loads**
Define dead and live service loads in separate load cases:
- click `Loads` at the top of the screen.
- click `New load` and type in "Dead" as the load case title.
- select `Element loads` in the side menu.
- select `Define` and and define a loads = 10 kN/m$^2$:

![Diagram of load case definition](image)
• repeat for a second load case titled "Live" with a uniform load = -3 kn/m² applied to all elements.
• click \texttt{1**solve} to solve the model.

\textbf{Results}

The slab deflections will be calculated according to Eurocode 2.
First we will check the \textit{STRAP} uncracked elastic deflection:

\textbf{Combinations:}

Three combinations are required
• ultimate loads - total - to calculate the reinforcement
• service loads - total - to calculate the immediate deflection $a_i$
• service loads - sustained - to calculate the long-term deflections $a_{i\text{t}}$; assume that 30\% of the live load is sustained.

To define the combinations:
• select \texttt{Combinations} in the side menu and \texttt{Define/rev}...
• define the following combinations:

<table>
<thead>
<tr>
<th>No</th>
<th>Title</th>
<th>1 Dead</th>
<th>2 Live</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ultimate</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>Service</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Sustained</td>
<td>1</td>
<td>0.3</td>
</tr>
</tbody>
</table>

\textbf{Elastic deflections:}

• click \texttt{General results} in the side menu and \texttt{Draw result}
• arrange the menu as follows, click \texttt{OK} ; the program displays the deflection contour map:

The maximum elastic deflection is 5.97 mm.
Deflections - cracked section:

- click [Slab deflection] in the side menu and specify the deflection parameters:

![Slab deflections parameters](image)

Notes:

1. The "creep factor" is used to calculate the total long-term deflection. The deflections calculated from the long-term combination using the effective moment-of-inertia are multiplied by this factor. The factor corresponds to:
   - Eurocode 2: Equation (7.20)
   - ACI 318: Equation (9-11)

2. The reinforcement values used to calculated the effective moments-of-inertia are determined as follows:
   - **Reinf. required for moments/forces**
     The program calculates the area required and then selects actual reinforcement according to the specified range of diameters and spacings. This actual area is used to calculate the effective moments-of-inertia.
   - **User defined reinforcement**
     The program uses the spacing and diameter specified in the reinforcement option in this dialog box for all elements, top and bottom, both directions. However, different reinforcement area may be defined for selected elements, as follows:
     - select **Options** in the side menu
     - select **[rein. para...]**

- click [Solve] to calculate the reinforcement, the effective moments-of-inertia and to solve the model again with the reduced stiffnesses.

- click [Display]
The maximum deflection is \( 19.8 \) mm, \( (19.8/5.97) = 3.3 \) times greater than the elastic deflection.

Estimate the deflection at node 93 in terms of \( L/x \), relative to support nodes 289 and 4:

- click \( \text{Display} \)
- select \( \text{Draw deflected shape} \) and click \( \text{OK} \); the program superimposes the deflected shape and deflection values:

![Deflected Shape Diagram]

- click \( \text{relative} \) at the bottom of the display.
- click on nodes 93, 289 and 4 (in that order). The program displays the relative deflection:

![Relative Deflection Table]

Display a table of the cracked section properties:

- click \( \text{Display} \)
select **Cracked sections table** and click **OK**; the program displays the following table:

<table>
<thead>
<tr>
<th>Elem.</th>
<th>Comb</th>
<th>Dir</th>
<th>Mcr</th>
<th>M</th>
<th>F</th>
<th>As</th>
<th>As'</th>
<th>x</th>
<th>Ir/Ig</th>
<th>le/Ig</th>
</tr>
</thead>
<tbody>
<tr>
<td>89</td>
<td>2</td>
<td>X</td>
<td>20.32</td>
<td>33.00</td>
<td>0.00</td>
<td>10</td>
<td>0</td>
<td>4.1</td>
<td>0.183</td>
<td>0.338</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
<td>19.92</td>
<td>18.09</td>
<td>0.00</td>
<td>6</td>
<td>0</td>
<td>3.3</td>
<td>0.120</td>
<td>1.000</td>
</tr>
<tr>
<td>90</td>
<td>2</td>
<td>X</td>
<td>20.23</td>
<td>32.30</td>
<td>0.00</td>
<td>9</td>
<td>0</td>
<td>4.0</td>
<td>0.170</td>
<td>0.332</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
<td>19.85</td>
<td>18.47</td>
<td>0.00</td>
<td>5</td>
<td>0</td>
<td>3.1</td>
<td>0.109</td>
<td>1.000</td>
</tr>
</tbody>
</table>

where:

- **Elem** - element number
- **Comb** - combination used for deflection calculation
- **Dir** - direction; properties are calculated in both reinforcement directions
- **Mcr** - the cracking moment
- **M** - the moment at the element center
- **F** - the axial force in the element
- **As** - the tension reinforcement (calculated, minimum or user-defined)
- **As'** - the compression reinforcement (calculated, minimum or user-defined)
- **x** - height of the compression block in the section
- **Ir/Ig** - ratio between the cracked and uncracked moments-of-inertia.
- **le/Ig** - ratio between the effective and uncracked moments-of-inertia.

For example, in element 90:

- the moment in the X-direction = 32.3 kN-m is greater than the cracking moment = 20.23 kN-m
- the section is cracked, hence the effective moment-of-inertia is 33.2% of the uncracked moment-of-inertia.
- in the Y-direction, the moment = 18.47 is less than the cracking moment, hence the program uses the uncracked section (le/Ig = 1.000)